CSCI 6114: Problem Set 5
Due: Wed Sep 31, 2026 11:59pm

September 24, 2026

1. Show that if a decision problem can be decided by (sufficiently) uniform NC circuits of depth d, then it can be solved by an algorithm using $O(d)$ work-space. In particular, conclude that $\mathsf{NC}^1 \subseteq \mathsf{L} = \mathsf{LogSpace}$. *Hint:* If you evaluate the circuit in a manner similar to depth-first search (rather than layer-by-layer, which is similar to breadth-first search), how many partial results do you need to store at once?

2. Show that Boolean Matrix Multiplication is in AC^0. Boolean Matrix Multiplication is the following problem: given two $n \times n$ matrices A, B with Boolean entries, compute the $n \times n$ Boolean matrix C whose entries are:
$$C[i,j] = \bigvee_{k=1}^{n} (A[i,k] \wedge B[k,j]) .$$

3. Show that $\mathsf{NL} \subseteq \mathsf{AC}^1$. *Hint:* Use the result from the previous question, the fact that the following problem is NL-complete under AC^0-reductions, and the fact that AC^1 is closed under AC^0-reductions:

   Directed s-t Connectivity
   Input: A directed graph G (say, by its adjacency matrix), and two vertices $s, t \in V(G)$
   Decide: Is there a directed path from s to t in G?

4. Show that HornSAT is P-complete under log-space reductions. You may use the fact that the Circuit Value Problem is P-complete. (Unrelated to the solution, but interesting to note: 2SAT is complete for nondeterministic log-space NL, which is much lower down the NC hierarchy: $\mathsf{NC}^1 \subseteq \mathsf{L} \subseteq \mathsf{NL} \subseteq \mathsf{AC}^1 \subseteq \mathsf{NC}^2 \subseteq \cdots \subseteq \mathsf{NC} \subseteq \mathsf{P}$.)